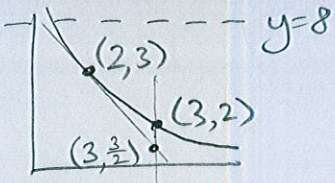
Consider the region defined by $y \le \frac{6}{x}$, $y \ge 6 - \frac{3}{2}x$ and $x \le 3$.



$x = \frac{6}{y}$ $x = \frac{2}{3}(6-y)$

SCORE: ____ / 14 PTS

[a] If the region is revolved around the line $y = 8$, write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid

$\frac{6}{x} = 6 - \frac{3}{2}x$

$12 = 12x - 3x^2$

$3x^2 - 12x + 12 = 0$

$3(x-2)^2 = 0$

$x = 2$

[i] using the disk or washer method **(NOTE: You do NOT need to simplify your integrand.)**

$\pi \int_2^3 \left(\left(8 - \left(6 - \frac{3}{2}x\right)\right)^2 - \left(8 - \frac{6}{x}\right)^2 \right) dx$ (1/2)

EXPLANATION: (1) FOR R
(1) FOR r
(1) FOR $R^2 - r^2$

★ (1) FOR EACH ITEM UNLESS OTHERWISE NOTED

[ii] using the shell method **(NOTE: You do NOT need to simplify your integrand.)**

$2\pi \left(\int_{\frac{3}{2}}^{2} (8-y)\left(3 - \frac{2}{3}(6-y)\right) dy + \int_2^3 (8-y)\left(\frac{6}{y} - \frac{2}{3}(6-y)\right) dy \right)$ (1/2) (1/2)

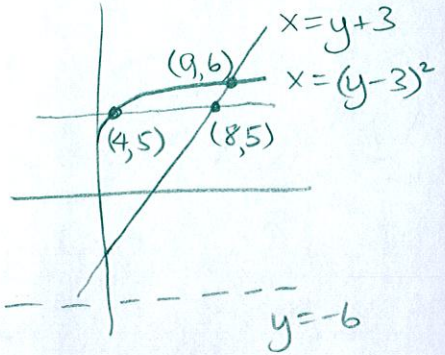
[b] Suppose the region is the base of a solid. Cross sections perpendicular to the x – axis are isosceles right triangles with their hypoteneuse in the base. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

$\frac{1}{4} \int_2^3 \left(\frac{6}{x} - \left(6 - \frac{3}{2}x\right)\right)^2 dx$ (1/2)

$A = \frac{1}{4}s^2$

The region bounded by $y = 3 + \sqrt{x}$, $y = x - 3$ and $y = 5$ is revolved around the line $y = -6$. **SCORE: ____ / 8 PTS**

Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid **using as few integrals as possible**.



$y+3=(y-3)^2$

$y+3=y^2-6y+9$

$0=y^2-7y+6$ (1)

$0=(y-1)(y-6)$

$y=1,6$

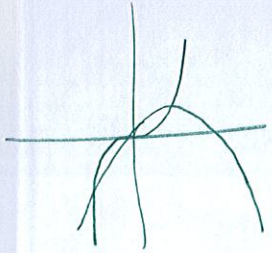
(1)

$2\pi \int_5^6 (y+6)(y+3-(y-3)^2)\,dy$

(2) (2) (2)

Find the area bounded by the curves $y = 4x^3$ and $y = 8x - 4x^2$.

NOTE: Your final answer must be a number, not an integral nor sum of integrals.



SCORE: ____ / 8 PTS

$4x^3 = 8x - 4x^2$

$4x^3 + 4x^2 - 8x = 0$

$4x(x+2)(x-1) = 0$

$x = -2, 0, 1$

$$\int_{-2}^{0} (4x^3 - (8x - 4x^2))\,dx + \int_{0}^{1} (8x - 4x^2 - 4x^3)\,dx$$ (1) EACH

$$= \left(x^4 - 4x^2 + \frac{4}{3}x^3\right)\Big|_{-2}^{0} + \left(-x^4 + 4x^2 - \frac{4}{3}x^3\right)\Big|_{0}^{1}$$

$$= 0 - \left(16 - 16 - \frac{32}{3}\right) + \left(-1 + 4 - \frac{4}{3}\right) - 0$$

$$= \frac{37}{3}$$